Verification of cryptographic protocols

From authentication to privacy

Steve Kremer

INRIA Nancy, LORIA

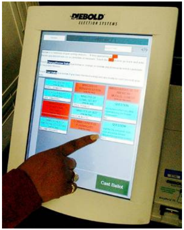
Séminaire Confiance Numérique

Cryptographic protocols everywhere!

- Distributed programs that
- use cryptographic primitives (encryption, digital signature ,. . .)
- to ensure security properties (confidentiality, authentication, anonymity,. . .)

E-commerce

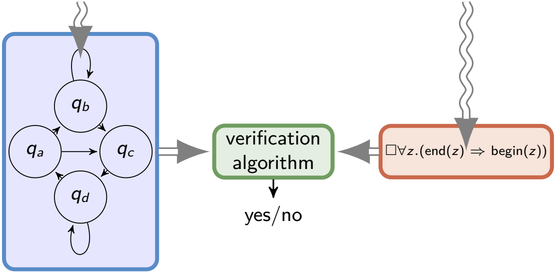
Mobile telephony



Electronic voting

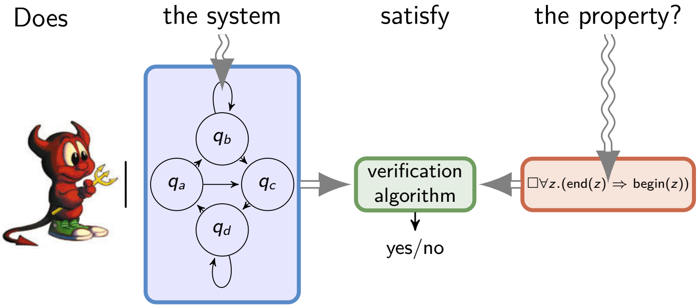
Formal verification of critical systems

Does the system satisfy the property?



Formal verification of critical systems

Applied to security protocols:



Difficulties :

$\rightsquigarrow$ arbitrary attacker controlling the network

$\rightsquigarrow$ infinite state system

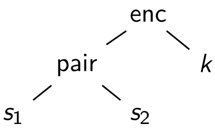
Techniques :

automated deduction, concurrency theory, model-checking, ...

Symbolic analysis

Symbolic techniques (following [Dolev&Yao'82]):

- messages = terms



- perfect cryptography (equational theories)

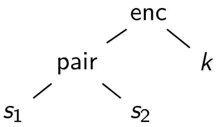
$$\mathrm{dec}(\mathrm{enc}(x,y),y) = x \quad \mathrm{fst}(\mathrm{pair}(x,y)) = x \quad \mathrm{snd}(\mathrm{pair}(x,y)) = y$$

- the network is the attacker

Symbolic analysis

Symbolic techniques (following [Dolev&Yao'82]):

- messages = terms



- perfect cryptography (equational theories)

$$\mathrm{dec}(\mathrm{enc}(x, y), y) = x \quad \mathrm{fst}(\mathrm{pair}(x, y)) = x \quad \mathrm{snd}(\mathrm{pair}(x, y)) = y$$

- the network is the attacker

Automated tools successfully found flaws in:

- Google's Single Sign-On protocol
- ISO/IEC 9798 standard for entity authentication
- commercial PKCS#11 key-management tokens
- ...

Automated verification?

Many good tools:

AVISPA, Casper, Maude-NPA, ProVerif, Scyther, Tamarin, . . .

Good at verifying trace properties (predicates on system behavior), e.g.,

- (weak) secrecy of a key
- correspondence properties

  If B ended a session with parameter p then A must have started a session with parameters p'.

Automated verification?

Many good tools:

AVISPA, Casper, Maude-NPA, ProVerif, Scyther, Tamarin, ...

Good at verifying trace properties (predicates on system behavior), e.g.,

- (weak) secrecy of a key
- correspondence properties

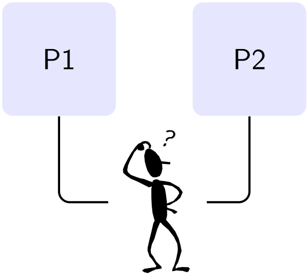
  If B ended a session with parameter p then A must have started a session with parameters p'.

Not all properties can be expressed on a trace.

$\rightsquigarrow$ recent interest in **indistinguishability properties**.

Indistinguishability (informally)

Can the adversary distinguish two situations, i.e. decide whether it is interacting with protocol P1 or protocol P2?



We write $P1 \approx P2$ when the adversary cannot distinguish $P1$ and $P2$

Indistinguishability in process calculi

Naturally modelled using equivalences from process calculi

e.g. [Spi calculus, Abadi & Gordon'96]
[Applied pi calculus, Abadi & Fournet'01]

Testing equivalence ($P \approx Q$)

for all processes A, we have that:

$$A \mid P \Downarrow c \text{ if, and only if, } A \mid Q \Downarrow c$$

$\longrightarrow$ $P \Downarrow c$ when P can send a message on the channel c.

Indistinguishability in process calculi

Naturally modelled using equivalences from process calculi

e.g. [Spi calculus, Abadi & Gordon'96]

[Applied pi calculus, Abadi & Fournet'01]

Testing equivalence ($P \approx Q$)

for all processes A, we have that:

$$A \mid P \Downarrow c \text{ if, and only if, } A \mid Q \Downarrow c$$

$\longrightarrow$ $P \Downarrow c$ when P can send a message on the channel c.

Remarks

- Process equivalences are well known notions in concurrency theory; much more difficult when adding support for crypto primitives
- A whole zoo of equivalences (with subtle differences)

A cryptographic process calculus

Protocols modelled in a process calculus, e.g. the applied pi calculus

$$
\begin{array}{rcll}
P ::= & & 0 & \\
& | & \mathrm{in}(c, x).P & \text{input} \\
& | & \mathrm{out}(c, t).P & \text{output} \\
& | & \text{if } t_1 = t_2 \text{ then } P \text{ else } Q & \text{conditional} \\
& | & P \parallel Q & \text{parallel} \\
& | & !P & \text{replication} \\
& | & \text{new } n.P & \text{restriction}
\end{array}
$$

A cryptographic process calculus

Protocols modelled in a process calculus, e.g. the applied pi calculus

$$
\begin{array}{rcll}
P ::= & & 0 & \\
& \mid & \mathrm{in}(c, x).P & \text{input} \\
& \mid & \mathrm{out}(c, t).P & \text{output} \\
& \mid & \text{if } t_1 = t_2 \text{ then } P \text{ else } Q & \text{conditional} \\
& \mid & P \parallel Q & \text{parallel} \\
& \mid & !P & \text{replication} \\
& \mid & \text{new } n.P & \text{restriction}
\end{array}
$$

Specificities:

- messages are **terms** (not just atomic names as in the pi calculus)
- equality in conditionals interpreted modulo an **equational theory**

Secrecy in symbolic models

In symbolic analysis secrecy is generally modelled as non-deducibility:

the attacker cannot compute the value of the secret

$\leadsto$ partial leakage is not detected

Example (Weak secrecy)

Let h be a one-way hash function. The protocol $P = \nu s.out(c, h(s))$ would be considered to enforce the secrecy of s.

Secrecy as indistinguishability

Stronger notions of secrecy can be defined using indistinguishability

- Strong secrecy of s: [Blanchet'04]

$$\mathbf{in}(c, \langle t_1, t_2 \rangle).\ P\{^{t_1}/_{s}\} \approx \mathbf{in}(c, \langle t_1, t_2 \rangle).\ P\{^{t_2}/_{s}\}$$

Even if the attacker chooses values t_1 or t_2 he cannot distinguish whether t_1 or t_2 was used as the secret.

- Resistance against offline guessing attacks (real-or-random): [Corin et al.'05]

$$P; \mathbf{out}(s) \approx P; \nu s'.\mathbf{out}(s')$$

The attacker cannot distinguish whether at the end of the protocol he is given the real secret or a random value.

How to model vote privacy?

How can we model "the attacker does not learn my vote (0 or 1)"?

How to model vote privacy?

How can we model "the attacker does not learn my vote (0 or 1)"?

- The attacker cannot learn the value of my vote

How to model vote privacy?

How can we model “the attacker does not learn my vote (0 or 1)”?

- The attacker cannot learn the value of my vote
  $\rightsquigarrow$ but the attacker knows values 0 and 1

How to model vote privacy?

How can we model “the attacker does not learn my vote (0 or 1)”?

- ~~The attacker cannot learn the value of my vote~~
- The attacker cannot distinguish when we change the voter identity: $V_A(v) \approx V_B(v)$

How to model vote privacy?

How can we model "the attacker does not learn my vote (0 or 1)"?

- ~~The attacker cannot learn the value of my vote~~

- The attacker cannot distinguish when we change the voter identity:
  $V_A(v) \approx V_B(v)$
  $\rightsquigarrow$ but identities are revealed

How to model vote privacy?

How can we model "the attacker does not learn my vote (0 or 1)"?

- ~~The attacker cannot learn the value of my vote~~
- ~~The attacker cannot distinguish when we change the voter identity~~: $\cancel{V_A(v) \approx V_B(v)}$
- The attacker cannot distinguish when change the vote: $V_A(0) \approx V_A(1)$

How to model vote privacy?

How can we model “the attacker does not learn my vote (0 or 1)”?

- ~~The attacker cannot learn the value of my vote~~
- ~~The attacker cannot distinguish when we change the voter identity:~~ $\cancel{V_A(v) \approx V_B(v)}$
- The attacker cannot distinguish when change the vote: $V_A(0) \approx V_A(1)$
  $\rightsquigarrow$ but election outcome is revealed

How to model vote privacy?

How can we model "the attacker does not learn my vote (0 or 1)"?

- ~~The attacker cannot learn the value of my vote~~
- ~~The attacker cannot distinguish when we change the voter identity~~: $\cancel{V_A(v) \approx V_B(v)}$
- ~~The attacker cannot distinguish when change the vote~~: $\cancel{V_A(0) \approx V_A(1)}$
- The attacker cannot distinguish the situation where two honest voters swap votes:

$$V_A(0) \parallel V_B(1) \approx V_A(1) \parallel V_B(0)$$

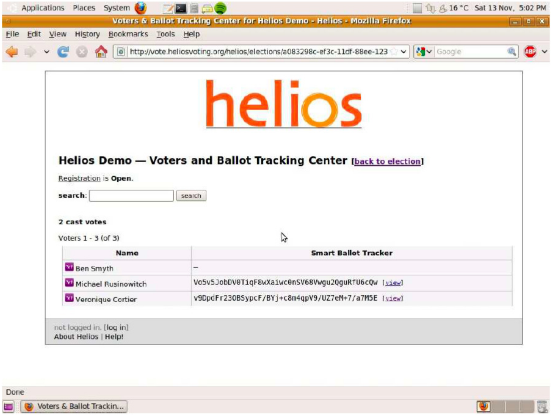
Also avoids the problematic case of unanimity!

[Kremer, Ryan '05]

The Helios e-voting protocol

Verifiable online elections via the Internet

http://heliosvoting.org/



Already in use:

- Election at Louvain University Princeton
- Election of the IACR board (major association in Cryptography)

Behavior of Helios (simplified)

Phase 1: voting

Bulletin Board		
Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_C\}_{pk(S)}$	$v_C = 0$ or 1

$pk(S)$: public key, the private key being shared among trustees.

Behavior of Helios (simplified)

Phase 1: voting

$$\xrightarrow{\{v_D\}_{pk(S)}}$$

Bulletin Board

Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_C\}_{pk(S)}$	$v_C = 0$ or 1

$pk(S)$: public key, the private key being shared among trustees.

Behavior of Helios (simplified)

Phase 1: voting

Bulletin Board

Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_C\}_{pk(S)}$	$v_C = 0$ or 1
David	$\{v_D\}_{pk(S)}$	$v_D = 0$ or 1

$pk(S)$: public key, the private key being shared among trustees.

Behavior of Helios (simplified)

Phase 1: voting

Bulletin Board

Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_C\}_{pk(S)}$	$v_C = 0$ or 1
David	$\{v_D\}_{pk(S)}$	$v_D = 0$ or 1
...	...	

Phase 2: Tallying using homomorphic encryption (El Gamal)

$$\prod_{i=1}^{n} \{v_i\}_{pk(S)} = \{\sum_{i=1}^{n} v_i\}_{pk(S)} \qquad \text{based on } g^a * g^b = g^{a+b}$$

$\rightarrow$ Only the final result needs to be decrypted!

$pk(S)$: public key, the private key being shared among trustees.

This is oversimplified!

Bulletin Board		
Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_C\}_{pk(S)}$	$v_C = 0$ or 1
David	$\{v_D\}_{pk(S)}$	
...	...	

Result: $\{v_A + v_B + v_C + v_D + \cdots\}_{pk(S)}$

This is oversimplified!

Bulletin Board		
Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_C\}_{pk(S)}$	$v_C = 0$ or 1
David	$\{v_D\}_{pk(S)}$	$v_D = 100$
...	...	

Result: $\{v_A + v_B + v_C + 100 + \cdots\}_{pk(S)}$

A malicious voter can cheat!

This is oversimplified!

Bulletin Board

Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_C\}_{pk(S)}$	$v_C = 0$ or 1
David	$\{v_D\}_{pk(S)}$	$\cancel{v_D = 100}$
...	...	

Result: $\{v_A + v_B + v_C + v_D + \cdots\}_{pk(S)}$

~~A malicious voter can cheat!~~

In Helios: use Zero Knowledge Proof

$$\{v_D\}_{pk(S)}, \mathrm{ZKP}\{v_D = 0 \text{ or } 1\}$$

A privacy attack on Helios

Bulletin Board

Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1

A privacy attack on Helios

Bulletin Board

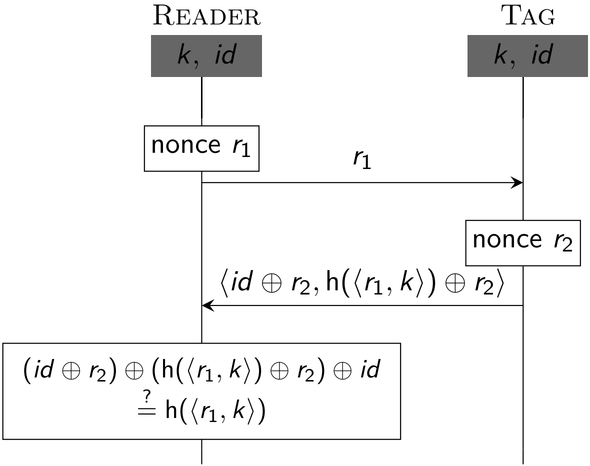
Alice	$\{v_A\}_{pk(S)}$	$v_A = 0$ or 1
Bob	$\{v_B\}_{pk(S)}$	$v_B = 0$ or 1
Chris	$\{v_A\}_{pk(S)}$	

Vote-copying attack:
copying Alice's vote introduces a bias in the outcome

Weakness in Helios discovered when trying to prove the previous definition of anonymity

[Cortier, Smyth '11]

Authentication protocol of a RFID tag



$$P_{\mathrm{tag}} = \mathbf{in}(c, x).\ \mathrm{new}\ r_2.\ \mathbf{out}(c, \langle id \oplus r_2,\ \mathsf{h}(\langle x, k\rangle) \oplus r_2\rangle).\ 0$$

Untraceability

An attacker must not be able to **link two sessions of a same tag**.

Modelled as an equivalence:

2 sessions of the **same** tag $\approx$ 2 sessions of **different** tags

Untraceability

An attacker must not be able to **link two sessions of a same tag**.

Modelled as an equivalence:

2 sessions of the **same** tag $\approx$ 2 sessions of **different** tags

Linkability attack:

$$
\begin{array}{rcl}
P_{\text{same}} & = & \mathbf{in}(c,x).\ \text{new } r_2.\ \mathbf{out}(c,t).\ \mathbf{in}(c,x).\ \text{new } r_2'.\ \mathbf{out}(c,t_{\text{s}}').0 \\
& \not\approx & \\
P_{\text{diff}} & = & \mathbf{in}(c,x).\ \text{new } r_2.\ \mathbf{out}(c,t).\ \mathbf{in}(c,x).\ \text{new } r_2'.\ \mathbf{out}(c,t_{\text{d}}').0
\end{array}
$$

where

$$
\begin{array}{rcl}
t & = & \langle id \oplus r_2,\ \mathsf{h}(\langle x, k\rangle) \oplus r_2 \rangle \\
t_{\text{s}}' & = & \langle id \oplus r_2',\ \mathsf{h}(\langle x, k\rangle) \oplus r_2' \rangle \\
t_{\text{d}}' & = & \langle id' \oplus r_2',\ \mathsf{h}(\langle x, k'\rangle) \oplus r_2' \rangle
\end{array}
$$

Untraceability

An attacker must not be able to **link two sessions of a same tag**.

Modelled as an equivalence:

2 sessions of the **same** tag $\approx$ 2 sessions of **different** tags

Linkability attack:

$$\begin{array}{rcl} P_{\mathsf{same}} & = & \mathbf{in}(c,x).\ \mathsf{new}\ r_2.\ \mathbf{out}(c,t).\ \mathbf{in}(c,x).\ \mathsf{new}\ r_2'.\ \mathbf{out}(c,t_{\mathsf{s}}').0 \\ & & \not\approx \\ P_{\mathsf{diff}} & = & \mathbf{in}(c,x).\ \mathsf{new}\ r_2.\ \mathbf{out}(c,t).\ \mathbf{in}(c,x).\ \mathsf{new}\ r_2'.\ \mathbf{out}(c,t_{\mathsf{d}}').0 \end{array}$$

where

$$\begin{array}{rcl} t & = & \langle id \oplus r_2,\ \mathsf{h}(\langle x, k\rangle) \oplus r_2\rangle \\ t_{\mathsf{s}}' & = & \langle id \oplus r_2',\ \mathsf{h}(\langle x, k\rangle) \oplus r_2'\rangle \\ t_{\mathsf{d}}' & = & \langle id' \oplus r_2',\ \mathsf{h}(\langle x, k'\rangle) \oplus r_2'\rangle \end{array}$$

distinguished by

$$(\mathsf{proj}_1(t) \oplus \mathsf{proj}_2(t) \stackrel{?}{=} \mathsf{proj}_1(t') \oplus \mathsf{proj}_2(t')$$

Our goals and approach for verifying equivalence properties

[Chadha, Ciobâcă, K., 2012]

. . . and actively developed since

Decision procedure for **trace equivalence**:

- many equational theories,
- practical implementation

Protocols modelled as **first order Horn clauses** (**bounded number of sessions**, i.e., no replication)

Resolution based procedure for trace equivalence for convergent equational theories (in particular **optimally reducing eq. theories**)

Terms and frames

Messages are modelled as first-order terms equipped with a convergent rewrite system R.

Secret values are modelled as names in a set $\mathcal{N}$.

We write $t =_{\mathsf{R}} u$ when $t\!\downarrow = u\!\downarrow$

Example

Signature: senc/3, sdec/2, pair/2, fst/1, snd/1, $\mathbf{0}$/0, $\mathbf{1}$/0

Rewrite system:
$\mathsf{sdec}(\mathsf{senc}(x, y, z), y) \rightarrow_{\mathsf{R}} x, \mathsf{fst}(\mathsf{pair}(x, y)) \rightarrow_{\mathsf{R}} x, \mathsf{snd}(pair(x, y)) \rightarrow_{\mathsf{R}} y$

Terms: $t_1 = \mathsf{senc}(n, k, r)$, $t_2 = \mathsf{sdec}(t_1, k) \quad (n, k, r \in \mathcal{N})$
We have that $t_2 =_{\mathsf{R}} n$

Deduction

Sequences of messages are grouped in a frame $\varphi = \{^{t_1}/_{w_1}, \ldots, ^{t_n}/_{w_n}\}$

What messages can an attacker compute?

Definition (Deduction)

A term t is *deducible from frame* φ *with a recipe* r ($\varphi \vdash^r t$) if $r\varphi =_{\mathsf{R}} t$ and r does not contain names in $\mathcal{N}$.

Example

Let $\varphi = \{^{\mathsf{senc}(n_1,k_1,r_1)}/_{w_1}, ^{\mathsf{senc}(n_2,k_2,r_2)}/_{w_2}, ^{k_1}/_{w_3}\}$.

We have that $\varphi \vdash^{\mathsf{sdec}(w_1,w_3)} n_1$, $\varphi \not\vdash n_2$, $\varphi \vdash^{\mathbf{1}} \mathbf{1}$

Static equivalence

Sequences of messages are grouped in a frame $\varphi = \{^{t_1}/_{w_1}, \ldots, ^{t_n}/_{w_n}\}$

Indistinguishability of sequences of messages

Definition (Static equivalence)

$(r_1 = r_2)\varphi$ if $\varphi \vdash^{r_1} t$ and $\varphi \vdash^{r_2} t$ for some t.

φ_1 *statically equivalent* to φ_2 $(\varphi_1 \approx_s \varphi_2)$ iff $(r_1 = r_2)\varphi_1 \Leftrightarrow (r_1 = r_2)\varphi_2$.

Examples

$$
\begin{array}{rcll}
\{^{n_1}/_{w_1}\} & \approx_s & \{^{n_2}/_{w_1}\} & \\
\{^{n_1}/_{w_1}, ^{n_2}/_{w_2}\} & \not\approx_s & \{^{n_1}/_{w_1}, ^{n_1}/_{w_2}\} & (w_1 \stackrel{?}{=} w_2) \\
\{^{\mathrm{senc}(\mathbf{0},k,r)}/_{w_1}\} & \approx_s & \{^{\mathrm{senc}(\mathbf{1},k,r)}/_{w_1}\} & \\
\{^{\mathrm{senc}(n,k,r)}/_{w_1}, ^{k}/_{w_2}\} & \not\approx_s & \{^{\mathrm{senc}(\mathbf{0},k,r)}/_{w_1}, ^{k}/_{w_2}\} & (sdec(w_1, w_2) \stackrel{?}{=} \mathbf{0})
\end{array}
$$

A simple crypto process calculus: syntax

Actions : $\mathbf{in}(c, x) \mid \mathbf{out}(c, t) \mid [s \stackrel{?}{=} t]$

Symbolic Trace: sequence of actions

Example

$$\begin{array}{rcl} T & = & \mathbf{out}(c, \mathrm{enc}(a, k)).\mathbf{out}(c, \mathrm{enc}(a', k)). \\ & & \mathbf{in}(c, x).\mathbf{out}(c, \mathrm{dec}(x, k)). \\ & & \mathbf{in}(c, y).[y \stackrel{?}{=} \mathrm{pair}(a, a')].\mathbf{out}(c, s) \end{array}$$

Process: set of symbolic traces

Remark: Parallel composition $(P \mid Q)$ can be defined as the set of interleavings

A simple crypto process calculus: semantics

Operational semantics: $(T, \varphi) \xrightarrow{\ell} (T', \varphi')$

$$\textsc{Receive}\ \frac{\varphi \vdash^r t}{(\mathbf{in}(c, x).T, \varphi) \xrightarrow{\mathbf{in}(c,r)} (T\{x \mapsto t\}, \varphi)}$$

$$\textsc{Test}\ \frac{s =_{\mathsf{R}} t}{([s \stackrel{?}{=} t].T, \varphi) \xrightarrow{\mathbf{test}} (T, \varphi)}$$

$$\textsc{Send}\ \frac{}{(\mathbf{out}(c, t).T, \varphi) \xrightarrow{\mathbf{out}(c)} (T, \varphi \cup \{w_{|dom(\varphi)|+1} \mapsto t\})}$$

$P \xrightarrow{\ell} (T', \varphi)$ if $\exists T \in P.\ (T, \emptyset) \xrightarrow{\ell} (T', \varphi)$

$\stackrel{\ell}{\Longrightarrow}$ if $\xrightarrow{\mathbf{test}^*\ \ell\ \mathbf{test}^*}$: weak semantics hiding silent test actions

Trace equivalences

Trace equivalence: $P \sqsubseteq_t Q$

if $(P, \emptyset) \xrightarrow{\ell_1, \ldots, \ell_n} (P', \varphi)$ then $\exists Q', \varphi'.\ (Q, \emptyset) \xrightarrow{\ell_1, \ldots, \ell_n} (Q', \varphi') \wedge \varphi \sim_s \varphi'$

$P \approx Q$ iff $P \sqsubseteq Q \wedge Q \sqsubseteq P$

Trace equivalences

Fine grained trace equivalence: $P \sqsubseteq_{ft} Q$
$\forall T \in P.\ \exists T' \in Q.\ T \approx_t T'$

Trace equivalence: $P \sqsubseteq_t Q$
if $(P, \emptyset) \xrightarrow{\ell_1,\dots,\ell_n} (P', \varphi)$ then $\exists Q', \varphi'.\ (Q, \emptyset) \xrightarrow{\ell_1,\dots,\ell_n} (Q', \varphi') \wedge \varphi \sim_s \varphi'$

$P \approx Q$ iff $P \sqsubseteq Q \wedge Q \sqsubseteq P$

Trace equivalences

Fine grained trace equivalence: $P \sqsubseteq_{ft} Q$

$\forall T \in P.\ \exists T' \in Q.\ T \approx_t T'$

$$\not\cap$$

Trace equivalence: $P \sqsubseteq_t Q$

if $(P, \emptyset) \xrightarrow{\ell_1, \ldots, \ell_n} (P', \varphi)$ then $\exists Q', \varphi'.\ (Q, \emptyset) \xrightarrow{\ell_1, \ldots, \ell_n} (Q', \varphi') \wedge \varphi \sim_s \varphi'$

$P \approx Q$ iff $P \sqsubseteq Q \wedge Q \sqsubseteq P$

Trace equivalences

Fine grained trace equivalence: $P \sqsubseteq_{ft} Q$

$\forall T \in P.\ \exists T' \in Q.\ T \approx_t T'$

$$\not\supseteq \;\; \cap$$

Trace equivalence: $P \sqsubseteq_t Q$

if $(P, \emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (P', \varphi)$ then $\exists Q', \varphi'.\ (Q, \emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (Q', \varphi') \wedge \varphi \sim_s \varphi'$

Coarse trace equivalence: $P \sqsubseteq_{ct} Q$

if $(P, \emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (P', \varphi) \wedge (r = s)\varphi$ then $\exists Q', \varphi'.\ (Q, \emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (Q', \varphi') \wedge (r = s)\varphi'$

$P \approx Q$ iff $P \sqsubseteq Q \wedge Q \sqsubseteq P$

Trace equivalences

Fine grained trace equivalence: $P \sqsubseteq_{ft} Q$
$\forall T \in P.\ \exists T' \in Q.\ T \approx_t T'$

$$\not\cap$$

Trace equivalence: $P \sqsubseteq_t Q$
if $(P, \emptyset) \xRightarrow{\ell_1,\ldots,\ell_n} (P', \varphi)$ then $\exists Q', \varphi'.\ (Q, \emptyset) \xRightarrow{\ell_1,\ldots,\ell_n} (Q', \varphi') \wedge \varphi \sim_s \varphi'$

$$\not\cap$$

Coarse trace equivalence: $P \sqsubseteq_{ct} Q$
if $(P, \emptyset) \xRightarrow{\ell_1,\ldots,\ell_n} (P', \varphi) \wedge (r = s)\varphi$ then $\exists Q', \varphi'.\ (Q, \emptyset) \xRightarrow{\ell_1,\ldots,\ell_n} (Q', \varphi') \wedge (r = s)\varphi'$

$P \approx Q$ iff $P \sqsubseteq Q \wedge Q \sqsubseteq P$

Trace equivalences

Fine grained trace equivalence: $P \sqsubseteq_{ft} Q$

$\forall T \in P.\ \exists T' \in Q.\ T \approx_t T'$

$\not\cap$

Trace equivalence: $P \sqsubseteq_{t} Q$

if $(P,\emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (P',\varphi)$ then $\exists Q',\varphi'.\ (Q,\emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (Q',\varphi') \wedge \varphi \sim_s \varphi'$

$\not\cap$ $\|$ det. proc.

Coarse trace equivalence: $P \sqsubseteq_{ct} Q$

if $(P,\emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (P',\varphi) \wedge (r=s)\varphi$ then $\exists Q',\varphi'.\ (Q,\emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (Q',\varphi') \wedge (r=s)\varphi'$

$P \approx Q$ iff $P \sqsubseteq Q \wedge Q \sqsubseteq P$

P is *determinate* if whenever $(P,\emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (T,\varphi)$ and $(P,\emptyset) \xrightarrow{\ell_1,\ldots,\ell_n} (T',\varphi')$ then $\varphi \approx_s \varphi'$.

Our procedure: overview

1. Model protocol and intruder capabilities in Horn clauses
2. Saturate clauses using dedicated resolution procedure
3. Check equivalence

Our procedure: overview

1. Model protocol and intruder capabilities in Horn clauses
2. Saturate clauses using dedicated resolution procedure
3. Check equivalence

We fail to verify trace equivalence (in general) :-(

- under-approximate trace equivalence ($\approx_{ft}$)
- over-approximate trace equivalence ($\approx_{ct}$)
- verify trace equivalence for determinate processes

1. Horn clause modelling: predicates

Predicates: interpreted over ground trace T

- Reachability predicate

  $T \models \mathsf{r}_{\ell_1,\ldots,\ell_n}$ if $(T, \emptyset) \xrightarrow{L_1} (T_1, \varphi_1) \xrightarrow{L_2} \ldots \xrightarrow{L_n} (T_n, \varphi_n)$ such that $\ell_i =_{\mathsf{R}} L_i\varphi_{i-1}$ for all $1 \leq i \leq n$

- intruder Knowledge predicate

  $T \models \mathsf{k}_{\ell_1,\ldots,\ell_n}(R, t)$ if $\mathsf{r}_{\ell_1,\ldots,\ell_n}$ then $\varphi_n \vdash^{R\sigma} t\sigma$

- Identity predicate

  $T \models \mathsf{i}_{\ell_1,\ldots,\ell_n}(R, R')$ if $\exists t.\ T \models \mathsf{k}_{\ell_1,\ldots,\ell_i}(R, t)$ and $T \models \mathsf{k}_{\ell_1,\ldots,\ell_i}(R', t)$

- Reachable Identity predicate

  $T \models \mathsf{ri}_{\ell_1,\ldots,\ell_n}(R, R')$ if $T \models \mathsf{i}_{\ell_1,\ldots,\ell_n}(R, R')$ and $T \models \mathsf{r}_{\ell_1,\ldots,\ell_n}$

1. Horn clause modelling: initial clauses

$$T = \mathbf{in}(c, x).[\mathrm{dec}(x, k) \stackrel{?}{=} a].\mathbf{out}(c, s)$$

Compute an initial set for trace T: seed(T)

$$\begin{array}{rcl}
\mathrm{r}_{\mathbf{in}(c,x)} & \Leftarrow & \mathrm{k}(X, x) \\
\mathrm{r}_{\mathbf{in}(c,x),\mathbf{test}} & \Leftarrow & \mathrm{k}(X, x), \mathrm{dec}(x, k) =_{\mathrm{R}} a \\
\mathrm{r}_{\mathbf{in}(c,x),\mathbf{test},\mathbf{out}(c)} & \Leftarrow & \mathrm{k}(X, x), \mathrm{dec}(x, k) =_{\mathrm{R}} a \\
 & & \\
\mathrm{k}_{\mathbf{in}(c,x),\mathbf{test},\mathbf{out}(c)}(w_1, s) & \Leftarrow & \mathrm{k}(X, x), \mathrm{dec}(x, k) =_{\mathrm{R}} a \\
 & & \\
\mathrm{k}_w(f(X_1, \ldots X_n), f(x_1, \ldots, x_k)) & \Leftarrow & \mathrm{k}_w(X_1, x_1), \ldots, \mathrm{k}_w(X_k, x_k) \\
 & & \text{for any function } f
\end{array}$$

1. Horn clause modelling: getting rid of equations

Use equational unification to remove tests:

$$\Big(H \Leftarrow B_1, \ldots, B_n, u =_{\mathrm{R}} v\Big) \quad \rightsquigarrow \quad \begin{array}{l} \Big((H \Leftarrow B_1, \ldots, B_n)\sigma_1\Big) \\ \ldots \\ \Big((H \Leftarrow B_1, \ldots, B_n)\sigma_k\Big) \end{array}$$

where $\sigma_1, \ldots, \sigma_k$ is a complete set of unifiers for $u =_{\mathrm{R}} v$.

1. Horn clause modelling: getting rid of equations

Use equational unification to remove tests:

$$\Big(H \Leftarrow B_1, \ldots, B_n, u =_{\mathrm{R}} v\Big) \quad \rightsquigarrow \quad \begin{array}{l} \Big((H \Leftarrow B_1, \ldots, B_n)\sigma_1\Big) \\ \ldots \\ \Big((H \Leftarrow B_1, \ldots, B_n)\sigma_k\Big) \end{array}$$

where $\sigma_1, \ldots, \sigma_k$ is a complete set of unifiers for $u =_{\mathrm{R}} v$.

Example

$$\mathsf{r}_{\mathbf{in}(c,x),\mathbf{test},\mathbf{out}(c)} \Leftarrow \mathsf{k}(X, x), \mathsf{dec}(x, k) =_{\mathrm{R}} a$$

$$\rightsquigarrow$$

$$\mathsf{r}_{\mathbf{in}(c,\mathsf{enc}(a,k)),\mathbf{test},\mathbf{out}(c)} \Leftarrow \mathsf{k}(X, \mathsf{enc}(a, k))$$

1. Horn clause modelling: getting rid of equations (2)

Use finite variant property ([Comon-Lund, Delaune'05]) to get rid of equational reasoning:

> *Finite variant property: possibility to precompute a finite set of all possible normal forms*

$$\Big(\mathsf{k}_h(R,t) \Leftarrow B_1,\ldots,B_n\Big) \quad \rightsquigarrow \quad \begin{array}{l} \Big((\mathsf{k}_H(R,t))\theta_1{\downarrow} \Leftarrow B_1\theta_1{\downarrow},\ldots,B_n\theta_1{\downarrow}\Big) \\ \ldots \\ \Big((\mathsf{k}_H(R,t))\theta_k{\downarrow} \Leftarrow B_1\theta_k{\downarrow},\ldots,B_n\theta_k{\downarrow}\Big). \end{array}$$

where $\theta_1,\ldots,\theta_k$ is a complete set of variants for t.

We can compute finite sets of variants and mgu_E for the class of optimally reducing theories (contains subterm convergent, blind sigs, td commitment, ...)

2. Saturation: goals of saturation

Saturate seed knowledge base using the following rules

$$\text{RESOLUTION}\ \dfrac{\begin{array}{c} f \in K, g \in K_{\text{solved}}, \qquad f = \Big(H \Leftarrow \mathsf{k}_{uv}(X,t), B_1, \ldots, B_n\Big) \\ g = \Big(\mathsf{k}_w(R,t') \Leftarrow B_{n+1}, \ldots, B_m\Big) \\ \sigma = \mathsf{mgu}(\mathsf{k}_u(X,t), \mathsf{k}_w(R,t')) \qquad t \notin \mathcal{X} \end{array}}{K := K \cup \Big((H \Leftarrow B_1, \ldots, B_m)\sigma\Big)}$$

$$\text{EQUATION}\ \dfrac{\begin{array}{c} f, g \in K_{\text{solved}}, \qquad f = \Big(\mathsf{k}_u(R,t) \Leftarrow B_1, \ldots, B_n\Big) \\ g = \Big(\mathsf{k}_{u'v'}(R',t') \Leftarrow B_{n+1}, \ldots, B_m\Big) \qquad \sigma = \mathsf{mgu}(\mathsf{k}_u(\_,t), \mathsf{k}_{u'}(\_,t')) \end{array}}{K = K \cup \Big((\mathsf{i}_{u'v'}(R,R') \Leftarrow B_1, \ldots, B_m)\sigma\Big)}$$

$$\text{TEST}\ \dfrac{\begin{array}{c} f, g \in K_{\text{solved}}, \\ f = \Big(\mathsf{i}_u(R,R') \Leftarrow B_1, \ldots, B_n\Big) \qquad g = \Big(\mathsf{r}_{u'v'} \Leftarrow B_{n+1}, \ldots, B_m\Big) \qquad \sigma = \mathsf{mgu}(u,u') \end{array}}{K = K \cup \Big((\mathsf{ri}_{u'v'}(R,R') \Leftarrow B_1, \ldots, B_m)\sigma\Big)}$$

2. Saturation rules: soundness, completeness, termination

A clause is solved if it is of the form

$$H \Leftarrow \mathsf{k}_{w_1}(X_1, x_1), \ldots, \mathsf{k}_{w_n}(X_n, x_n)$$

2. Saturation rules: soundness, completeness, termination

A clause is solved if it is of the form

$$H \Leftarrow \mathsf{k}_{w_1}(X_1, x_1), \ldots, \mathsf{k}_{w_n}(X_n, x_n)$$

- Sound: If $f \in \mathsf{sat}(\mathsf{seed}(T))$ then $T \models f$

2. Saturation rules: soundness, completeness, termination

A clause is solved if it is of the form

$$H \Leftarrow \mathsf{k}_{w_1}(X_1, x_1), \dots, \mathsf{k}_{w_n}(X_n, x_n)$$

- Sound: If $f \in \mathsf{sat}(\mathsf{seed}(T))$ then $T \models f$
- Complete: If $(T, \emptyset) \xrightarrow{L_1,\dots,L_n} (S, \varphi)$ and $K = \mathsf{sat}(\mathsf{seed}(T))_{\mathsf{solved}}$ then
  1. $\mathsf{r}_{L_1,\dots,L_n}$ is a consequence of K
  2. if $\varphi \vdash^R t$ then $\mathsf{k}_{L_1,\dots,L_n}(R, t\downarrow)$ is a consequence of K
  3. if $\varphi \vdash^R t$ and $\varphi \vdash^{R'} t$, then $\mathsf{i}_{L_1,\dots,L_n}(R, R')$ is a consequence of K

2. Saturation rules: soundness, completeness, termination

A clause is solved if it is of the form

$$H \Leftarrow \mathsf{k}_{w_1}(X_1, x_1), \ldots, \mathsf{k}_{w_n}(X_n, x_n)$$

- Sound: If $f \in \mathsf{sat}(\mathsf{seed}(T))$ then $T \models f$
- Complete: If $(T, \emptyset) \xrightarrow{L_1,\ldots,L_n} (S, \varphi)$ and $K = \mathsf{sat}(\mathsf{seed}(T))_{\mathsf{solved}}$ then
  1. $\mathsf{r}_{L_1,\ldots,L_n}$ is a consequence of K
  2. if $\varphi \vdash^R t$ then $\mathsf{k}_{L_1,\ldots,L_n}(R, t\!\downarrow)$ is a consequence of K
  3. if $\varphi \vdash^R t$ and $\varphi \vdash^{R'} t$, then $\mathsf{i}_{L_1,\ldots,L_n}(R, R')$ is a consequence of K
- Termination:
  - guaranteed for subterm convergent equational theories;
  - in practice terminates also on examples outside this class.

3. Checking equivalence

To check that $T \sqsubseteq_{ct} Q$

1. saturate: let $K = \mathsf{sat}(\mathsf{seed}(T))_{\mathsf{solved}}$
2. check reachability:
   for each $\mathsf{r}_{L_1,\ldots,L_n} \Leftarrow \mathsf{k}_{h_1}(X_1, x_1), \ldots \mathsf{k}_{h_k}(X_k, x_k) \in K$
   check that $Q, \emptyset \xrightarrow{L_1,\ldots,L_n} Q', \varphi$
3. check equalities:
   for each $\mathsf{ri}_{L_1,\ldots,L_n}(R_1, R_2) \Leftarrow \mathsf{k}_{h_1}(X_1, x_1), \ldots \mathsf{k}_{h_k}(X_k, x_k) \in K$
   check that $Q, \emptyset \xrightarrow{L_1,\ldots,L_n} Q', \varphi$ and $(R_1 = R_2)\varphi$

The AKISS tool

AKISS

(Active Knowledge In Security protocolS)

`https://github.com/akiss`

Examples:

- Strong secrecy
  NSL protocol and Blanchet's variant's of Denning-Sacco (det. processes)
- Resistance to offline guessing attacks
  EKE (det. process)
- (Everlasting) Vote privacy: FOO, Okamoto, Helios and Moran-Naor electronic voting protocols
- **New**: support for $\oplus$ (RFID protocols)

Conclusions

- Process equivalences are the **main tool to model security properties** (except authentication)
- **Theoretical understanding** still rather poor: decidability for which equational theory? Complexity?
- **Tool support** not yet mature enough
  - AKISS: no else branches, approximates trace equivalence
  - APTE: only fixed equational theory (encryption, signature, hash)
  - ProVerif: unbounded number of sessions, but false attacks may occur
- WIP: a new procedure that takes **the best of both AKISS and APTE**:

  real trace equivalence + else branches + many equational theories